AP Calculus BC

Inverse Functions

1) $f(x) = x^3 + 7x + 2$

$f(1) = 10$	$g(10) = 1$
$f'(x) = 3x^2 + 7$ $f'(1) = 10$	$g'(10) = \frac{1}{10}$

2) $f(x) = x^5 + 3x^3 + 7x + 2$

$f(1) = 13$	$g(13) = 1$
$f'(x) = 5x^4 + 9x^2 + 7$ $f'(1) = 21$	$g'(13) = \frac{1}{21}$

3) $f(x) = 7(x+1)^3 + \sin^3 x$

$f(0) = 7$	$g(7) = 0$
$f'(x) = 21(x+1)^2 + 3\sin^2 x \cos x$ $f'(0) = 21$	$g'(7) = \frac{1}{f'(0)} = \frac{1}{21}$

4) $f(x) = x^7 + 2x + 9$

$f(1) = 12$	$g(12) = 1$
$f'(x) = 7x^6 + 2$ $f'(1) = 9$	$g(12) = \frac{1}{f'(1)} = \frac{1}{9}$

5) $g(x) = f^{-1}(x)$

$f(1) = 3$	$g(3) = 1$
$f'(1) = -5$	$g'(3) = -\frac{1}{5}$

6) $g(x) = f^{-1}(x)$

$f(2) = 4$	$g(4) = 2$
$f'(2) = 7$	$g'(4) = \frac{1}{7}$

7) $y = \arcsin(3x+2)$

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-(3x+2)^2}} = 3\left(1-(3x+2)^2\right)^{1/2}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2}\left[1-(3x+2)^2\right]^{-1/2} \cdot \left(-2(3x+2)\cdot 3\right)$$

8) $y = \arctan(\cos x)$

$$y' = \frac{-\sin x}{1+\cos^2 x}$$

$$y'' = \frac{(1+\cos^2 x)(-\cos x) - \sin^2 x \cdot 2\cos x}{(1+\cos^2 x)^2}$$

$$y''\Big|_{x=\frac{\pi}{3}} = \frac{(1+\frac{1}{4})(-\frac{1}{2}) - (\frac{3}{4})(1)}{(1+\frac{1}{4})^2}$$